Towards a Unified Model of Chords in Western Harmony

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Abstract

Chord-based harmony is an important aspect of many types of Western music, across genres, regions, and historical eras. However, the consistent representation and comparison of harmony across a wide range of styles (e.g. classical music, Jazz, Rock, or Pop) is a challenging task. Moreover, even within a single musical style, multiple theories of harmony may exist, each relying on its own (possibly implicit) assumptions and leading to harmonic analyses with a distinct focus (e.g. on the root of a chord vs. its bass note) or representation (e.g. spelled vs. enharmonic pitch classes). Cross-stylistic comparisons (as well as comparisons within a single style involving multiple annotation systems) are therefore even more difficult, particularly in a large-scale computational setting that requires a common overarching representation. To address these problems, we propose a model which allows for the representation of chords at multiple levels of abstraction: from chord realizations on the score level (if available), to pitch-class collections (including a potential application of different equivalences, such as enharmonic or octave equivalence), to pitch- and chord-level functions and higher-order abstractions. Importantly, our proposed model is also well-defined for theories which do not specify information at each level of abstraction (e.g., some theories make no claims about harmonic function), representing only those harmonic properties that are included and inducing others where possible (e.g., deriving scale degrees from root and key information). Our model thus represents an important step towards a unified representation of harmony and its various applications.

Introduction

Harmony constitutes an essential aspect of many Western musical styles [1,2,3,4,5,6,7]. There are a large number of harmonic annotation systems in music theory and analysis, including absolute chord labels, Roman numerals, Riemannian functional symbols [8,9], pitch-class sets [10,11,12], and Tonfeld labels [13,14]. Such annotation systems provide sets of symbols attributing properties to a segment of music, which relate that segment to an underlying harmonic theory [15]. Harmonic labels can be assigned manually or algorithmically, and the systems make different assumptions about harmonic and non-harmonic notes, underlying tonality, the level of abstraction, and the style in focus.

In recent years, several formal annotation standards were proposed for digital corpus research and Music Information Retrieval (MIR) [16,17,18,19,20,21,22,23,24,25,26,27,28,29,30, 31] (see also [32] for a recent overview and discussion). These standards were designed for different purposes, or relate to different styles of music, and therefore encode different sets of features (e.g., a standard for encoding chord progressions in rock music would reasonably not include the specificity of one designed for functional harmony annotations or chord extensions found in jazz). Despite this diversity, however, many systems overlap in a substantial number of the features they express.

For our purpose of an overarching harmonic notation and encoding system, as well as for the practical sake of musicological cross-style or diachronic analyses and comparisons, it is necessary to have available a unified representation for diverse harmonic practices. Along these lines, the most relevant standard is the absolute chord syntax proposed by [19], which has been widely adopted in MIR. Here, roots are explicitly stated as (spelled) pitch classes, and the bass and the upper interval structure of the chord are given as (specific) intervals relative to the root (e.g., a Cmin7/Eb chord is represented as "C: (b3, 5, b7) / b3"). They are thus independent of underlying key or scale contexts. This annotation standard forms the basis of several formal ontologies [33,34], which represent chords as graphs and most closely resemble our general proposal here. The objective of this paper is to propose a unified representation of chords for the comparative purpose outlined above, making it possible to characterize and query joint, translatable, and different features across standards.

1 Problem setting

The different assumptions for distinct systems of harmonic analysis define the problem setting for our unified chord model. Fig. 1 shows a modern engraving of Arcangelo Corelli's *Sonata a tre*, op. 1, no. 8, *Largo*, mm. 7–12 (1681 princeps edition), along with several harmonic annotation systems underneath, specifically figured bass (taken from the print) with bass degrees [35], Roman numerals, Riemannian function symbols [36], and absolute chords. Each of these annotation systems expresses the musical harmony from its own perspective, encoding some features explicitly, others implicitly, and others not at all. A universal chord model must be able to encompass all of these systems and more.

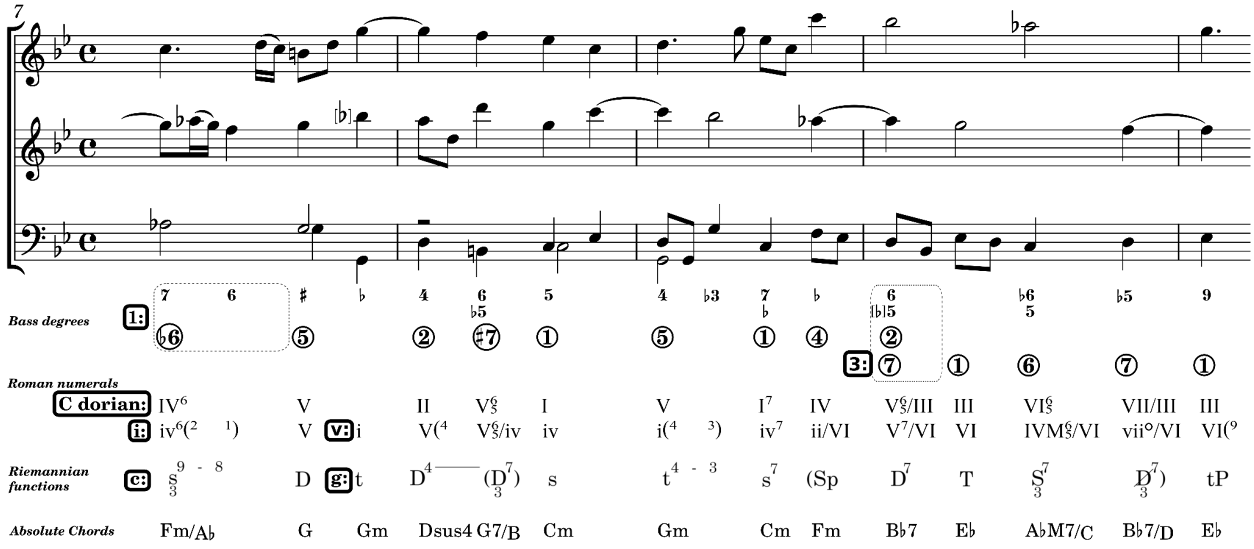

Figure 1: Arcangelo Corelli, *Sonatina a tre*, op. 1, no. 8 "Largo", mm. 7–11.

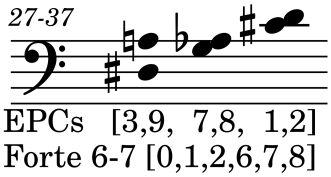

Figure 2: S. Gubaidulina, *String Trio* (1988), hexachord in mm. 27–37 (right).

Another challenge for an overarching model are chord annotations stemming from non-diatonic or non-triadic musical contexts, or from settings where diatonic chords cannot be labeled other than enharmonically (e.g. in a MIDI file). Fig. 2 shows an example of a non-triadic chord from Sofia Gubaidulina's *String Trio* (shown as reduction). It is the only chord within a segment of roughly ten bars, and it is played homophonically by the three instruments without rests. This chord defies analytical categories linked to triadic music, such as the root of a stack of thirds. The central note of the chord is D4, since the passage begins with a D4 unison in all instruments and builds gradually in a downward manner. After the introduction of our chord model, section 3 exemplifies how such a challenging case can be represented.

2 A unified chord model

Our proposed model represents chords as graphs, where the chord label and its position in a piece form a central node, and properties of the chord are labeled edges and attached nodes. A formal definition of our model, a comparison of several harmonic annotation standards, as well as more example graphical chord diagrams, can be found in our supplementary online material.[1] Graphically, Fig. 3a shows how the model represents the first chord from the Corelli example in Fig. 1. Here, categorical properties are represented by white rectangles, the key by a green parallelogram, and pitches and octaves are given in red and white circles, respectively.

Fundamentally, the model is based on viewing chord labels as selections of pitch classes (PCs, red circles in Fig. 3a) plus octaves. To account for the different chord encodings described

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[1] https://github.com/DCMLab/chord-model

above, we model different types of pitch classes: Generic Pitch Classes (GPCs; A-G), Spelled Pitch Classes (SPCs; GPC plus accidentals), and Enharmonic Pitch Classes (EPCs; MIDI note number mod 12). An SPC can be converted into an EPC or a GPC, but not vice versa.

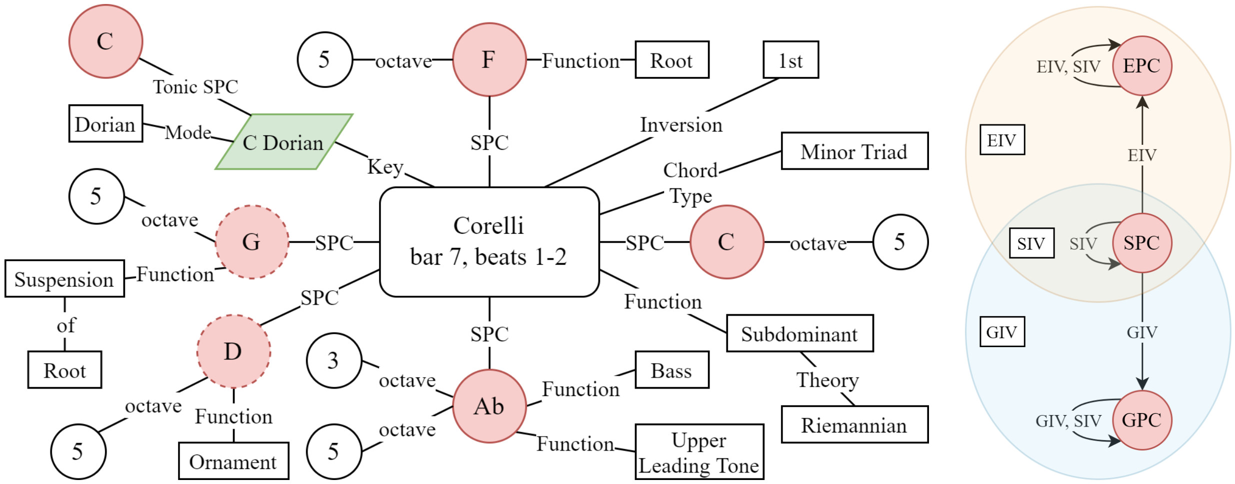

Figure 3. a) Our model's full representation of the first chord from the Corelli example in Fig. 1 (left). b) The different pitch types in our model, together with the result of transposing them by some interval type (the arrows), and the interval type that measures the distance between two pitch types (the labeled ovals).

Each of a chord's PCs can also be represented as the interval above some reference PC (e.g. the chord's root or bass). Analogous to PCs, these intervals may be generic (GIV; the difference between two GPCs, e.g. any 3rd), specific (SIV; the difference between two SPCs, e.g. a major 3rd), or enharmonic (EIV; the difference between two EPCs, e.g. 5 semitones). Fig. 3b illustrates the relationships between pitch and interval types in our model. Arrows indicate the resulting pitch type when transposing some PC by an interval (e.g., an SPC transposed by a GIV results in a GPC), and the large ovals specify the resulting interval type when measuring the distance between two PCs (e.g., the distance between a GPC and an SPC results in a GIV). These transformations are useful both for viewing a chord's pitches as intervals, and for calculating PCs given a chord label that denotes only intervals. Similarly, the model can represent a chord's pitches as scale degrees (SDs) of a mode. If the key (essentially a set of SIVs with a reference tonic PC) is known, an SD is equivalent to an SIV over the reference tonic PC (see Relative Pitch Classes below for more details). In what follows, we describe different aspects of the model using the first chord from the Corelli example (Fig. 1).

Score level. This least abstract representation of a chord consists of the set of pitches that are taken from all the notes within the segment referred to by the chord symbol. On this level, pitches are typically represented as SPCs, although we can also model annotated MIDI files at this level with pitches viewable only as EPCs. Each PC is also associated with one or multiple octaves. This representation allows the model to group pitches into pitch classes.

Pitch equivalences and missing notes. The model allows for abstraction from the score level by applying different equivalence operations, e.g., octave and enharmonic equivalence. To apply octave equivalence, the octave(s) of a pitch are simply ignored. This allows one to interpret a chord as the set of unique PCs occuring in the chord: a pitch-class set. In cases where octave information is not available because a symbolic pitch representation is missing or does not include all pitches expressed by a chord label, octave equivalence is therefore necessarily assumed. Enharmonic equivalence is represented as a flag that may be associated either with individual PCs or the entire chord, converting the corresponding PCs to EPCs. Conversely, pitch classes and complete chords that are expressed as EPCs only (e.g., those from MIDI files), by default, come with an enharmonic equivalence flag.

Pitch functions. PCs can be assigned functions within the chord. Importantly, each PC can be classified as either a chord tone or a non-chord tone. The possibility of ignoring non-chord tones, such as suspensions or ornaments, is common to many annotation standards. Other common pitch functions are, for example, root, bass note, and leading tone, but this set of categorical pitch functions can easily be extended.

Relative pitch classes. All absolute PCs can be expressed in relation to a particular PC, and/or to an ordered collection of intervals such as a mode or a Tonfeld. In the following, we refer to any ordered collection of SIVs as a "Mode", and the combination of a mode and a tonic pitch as a "Key", represented in our graphs as a green parallelogram. The tonic of a key, if present, may be represented as an absolute PC, or again relative to another tonic or key, e.g., for secondary dominants and other chord borrowings, or for indicating a local key. Various levels of a tonal hierarchy may be disambiguated by a "Type" feature on the key. If viewed relative to a given tonic SPC, every PC type (GPC, SPC, EPC) can be represented through its corresponding interval type (GIV, SIV, EIV), relative to this PC. In cases where the mode defines exactly one SIV for each GPC (e.g. major, natural/harmonic minor, phrygian, hypodorian, etc.), they are commonly represented as generic scale degrees (GSDs; 1-7, starting from the tonic, a one-to-one mapping from GPCs to SIVs). In these cases, each PC can also be expressed as a specific scale degree (SSD; GSD plus accidentals). Conversely, if a chord label expresses its PC content through GSDs (e.g., the generic Roman numerals in Fig. 1), or SSDs (e.g., the specific Roman numerals and the bass degrees), key information is required to convert the scale degrees into PCs. In the case of other modes (e.g., octatonic, hexatonic, and pentatonic scales), the relative representation of chordal PCs as SDs is not common and, in our model, corresponding relative PCs remain defined in terms of intervals.

Chord functions and properties. Chord-level properties in addition to key information or enharmonic equivalence may also be added to each chord. These include chord type, inversion, and chord function (e.g., tonic, dominant, predominant); custom functions and properties may also be defined and used. To disambiguate functions that occur in multiple theories, a chord function can also have a "Theory" property indicating the particular theory through which the function has been assigned. This allows, e.g., dominant chords from Tonfeld and Riemannian theories to be grouped together (if the theory property is ignored), or expressed separately.

3 Applications

Fig. 4 shows the graph for the post-tonal example in Fig. 2. Here, we represent the SPCs of the chord together with their EPCs and attach the "Central tone" function to D as provided by an analyst for the musical passage.

Since our model is represented as a graph, musicological research questions can be reformulated as graph queries, where all chords with a matching graph structure are valid answers to such a query. Fig. 5 shows the corresponding graph for a possibly diachronic or cross-stylistic example query, where the queried element is represented as a question mark. Using the model's flexibility, and its various possible representations of PCs, formulating queries such as this on a wide range of annotations would become a straightforward task, regardless of their underlying representations.

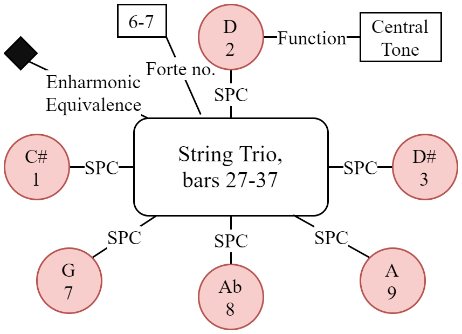

Figure 4. Representation of the chord in Fig. 2. Notice that the EPC representation is present for each SPC. The corresponding Forte number has been included as a chord property.

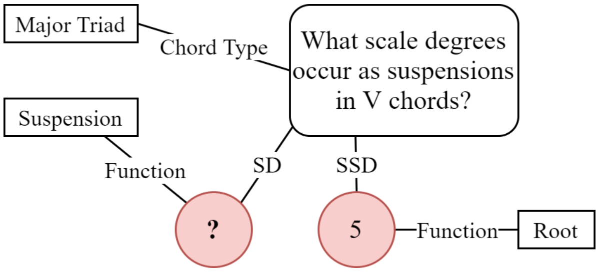

Figure 5. An example graph query using our model.

Conclusion

We proposed a unified chord model and showed that it can generalize over a number of existing harmonic annotation standards and is capable of expressing a variety of challenging analytical cases in a wide range of styles including Western classical, late-Romantic, Jazz, and contemporary post-tonal music. While the model may not be exhaustive, its general and flexible nature ensures its extensibility: its only requirement is that ‘chord’ in the sense of a collection of pitches is a meaningful concept in this style. It is therefore well-suited for music encoding standards that seek to consistently represent a broad variety of annotation systems, potentially allowing for conversion between them. Since the landscape of formal representations for harmony is so broad, we argue for a move towards a generalized standard for virtually all harmonic phenomena, which flexibly combines the utilities of each approach when possible. Our contribution should be understood as a first step towards this goal.

Acknowledgements

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